Influence of random bulk inhomogeneities on quasioptical cavity resonator spectrum

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The statistical spectral theory of oscillations in a quasioptical cavity resonator filled with random inhomogeneities is suggested. It is shown that inhomogeneities in the resonator lead to intermode scattering which results in the shift and broadening of spectral lines. The shift and the broadening of each spectral line is strongly depended upon the frequency distance between the nearest-neighbor spectral lines. As this distance increases, the influence of inhomogeneities is sharply reduced. Solitary spectral lines that have quite a large distance to the nearest neighbors are slightly changed due to small inhomogeneities. Owing to such a selective influence of inhomogeneities on spectral lines the effective spectrum rarefaction arises. Both the shift and the broadening of spectral lines as well as spectrum rarefaction in the quasioptical cavity millimeter wave resonator were detected experimentally. We have found out that inhomogeneities result in the resonator spectrum stochastization. As a result, the spectrum becomes composite, i.e., it consists of both regular and random parts. The active self-excited system based on the inhomogeneous quasioptical cavity millimeter wave resonator with a Gunn diode was examined as well. The inhomogeneous quasioptical cavity millimeter wave resonator (passive and active) can serve as a model of a semiconductor quantum billiard. Based on our results we propose that such a billiard with the spectrum rarefied by random inhomogeneities be used as an active semiconductor laser system.

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I. INTRODUCTION

The electromagnetic wave propagation in random inhomogeneous media has been the issue of the day for several decades. Numerous publications are devoted to the analysis of different aspects of this problem (see Refs. [1,2] and references therein). In this scientific area the subject of research is normally the scattering of electromagnetic (acoustic) waves in unbounded or partially confined systems (for example, in waveguides) which contain random inhomogeneities. Researches on wave propagation in open statistically irregular systems are stimulated by numerous applications for long-distance signal transmission in both radio and optical wave ranges.

At present the electromagnetic oscillations in confined resonant systems with random inhomogeneities, in particular, quasioptical microwave resonators, are also under study. Up to date, however, the satisfactory solution to this problem was not found both in theoretically and experimentally. On the one hand, the well-elaborated theories of wave propagation in disordered media use the statistical isotropy and the scattering potential homogeneity conditions [1,2], which basically cannot be implemented for confined systems. On the other hand, the random matrix theory (RMT) that is commonly used to analyze the confined systems [3,4] also has significant limitations. For RMT application it is necessary to express the Hamiltonian of the system in terms of a matrix whose all elements are random. Such a matrix can belong, for instance, to the ensemble of Gaussian orthogonal matrices (GOE). In this case matrix elements are real, symmetrical to the time inversion, and invariant to orthogonal transformations. The system with such a Hamiltonian is not integrable and the motion in it is completely chaotic. The examples of the completely chaotic systems are microwave resonators similar to the Sinai and Bunimovich billiards. Their chaotic spectra are well described by RMT [4–6]. The quasioptical cavity resonator even with small random inhomogeneities being considered in the present paper belongs to neither an integrable system nor a completely chaotic one. In this system we found that by inserting the inhomogeneities into the resonator its spectrum becomes mixed, i.e., it contains both regular and chaotic components simultaneously. Therefore, strictly speaking, to study the spectrum we cannot use the RMT approach [4] so we need a different one. Experimental study of electromagnetic oscillations spectrum in a quasioptical cavity resonator filled with inhomogeneities also requires new techniques at wide frequency range, including millimeter waves.

In our recent paper [7] we suggested a new spectral approach to studying confined systems with random inhomogeneities. Spectral properties of spherical quasi-optical millimeter wave cavity resonator filled with random sapphire particles were considered in the above-mentioned paper. The sapphire particles having dimensions of the order of an operating wavelength affect significantly the resonator spectrum. The spherical frequency degeneracy is completely removed and the spectral lines have the chaotic distribution on a frequency axis. The lines became wider and the quality factor was correspondingly decreased. The analogous broadening of spectral lines caused by random inhomogeneities has been detected in Refs. [8,9], where the resonators with random rough boundaries were studied.

Recently, the influence of random inhomogeneities on the resonator spectrum has attracted a great deal of attention in connection with the design of lasers on open microresonators [10]. In such resonators, the whispering-gallery modes with superhigh quality factors can be excited. The possibility to

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realize these quality factors strongly depends on the amount of inhomogeneities.

The cavity resonator with small dissipation loss is an almost Hermitian system whose oscillations are set up due to the restricted motion of electromagnetic waves. Upon introducing random elastically scattering inhomogeneities into the bulk of the resonator its spectrum should remain almost discrete, with an accuracy governed by the level of dissipation. However, as it was detected in the experiment [7], the broadening of spectral lines in a randomly filled resonator can go far beyond the value prescribed by the ohmic loss in the system. In this context a question arises: what is the overriding physical mechanism for spectral lines broadening and spectrum stochastization in random inhomogeneous resonators? The analysis of this issue is one of the goals of the present study.

In the present paper, the physical nature of the broadening and the shift of spectral lines of the quasioptical cavity resonator filled with randomly distributed bulk inhomogeneities has been studied both theoretically and experimentally. We have elaborated the original spectral theory based on the mode separation technique. The technique previously developed in Refs. [11,12] for open waveguide-type systems is extended here to closed systems, in particular, to a cylindrical cavity resonator. The proposed method applied directly to the master dynamic equation of the problem enables one to identify the principal physical mechanism of unexpectedly large spectral lines broadening with nondissipative intermode scattering. This type of scattering causes the width of nearest-neighboring spectral lines to increase more intensely than the width of solitary spectral lines. Owing to this fact the quality factor and, respectively, the intensity of the nearest-neighboring spectral lines tend to decrease sharply, but these parameters of solitary lines are only slightly changed. Below we will refer to such a selective change in the spectral lines as the “spectrum rarefaction.”

In order to verify theoretical predictions, systematic measurements of the quasioptical cavity resonator spectrum were performed with different realizations of random infill. We detected the predicted rarefaction effect and proved its origin to be related to the intermode scattering on non-dissipative random inhomogeneities. The influence of such inhomogeneities can be estimated in two ways. On the one hand, the broadening of spectral lines is normally perceived as a negative effect for resonance systems. But, on the other hand, as far as the lasing properties of the cavity resonator are concerned the spectrum rarefaction caused by random inhomogeneities can be viewed as a positive property.

One more aspect of the effect of random inhomogeneities on the resonator spectrum is related to its stochastization. Below we carry out statistical analysis of the resonator spectrum with a different amount of inhomogeneities. The interfrequency (IF) interval distribution in the spectrum of the resonator with a small number of inhomogeneities appears to be close to the Poisson distribution. This kind of distribution is typical for the systems with noncorrelated IF intervals. Highly chaotic part of the spectrum gradually appears when the number of inhomogeneities becomes sufficiently large. In the general case the spectrum is mixed, i.e., it contains both regular and chaotic parts. We estimate the relationship between regular and chaotic parts of the spectrum by making the statistical analysis of the IF intervals as a function of the number of inhomogeneities in the resonator. We also carried out the electromagnetic modeling of a semiconductor laser having random inhomogeneities. To this end, we used the quasioptical millimeter wave cavity resonator filled with random inhomogeneities and inserted the Gunn diode as an active element. In this system the conditions for self-excitation were studied. The generation was found to be an unstable and multifrequency near the excitation threshold. It can be explained by frequency jumps in the dense spectrum of the resonator without inhomogeneities. As the resonator is filled with randomly distributed inhomogeneities, the multifrequency generation disappears. Near the excitation threshold noise generation was detected whereas far from the threshold single frequency stable generation was observed.

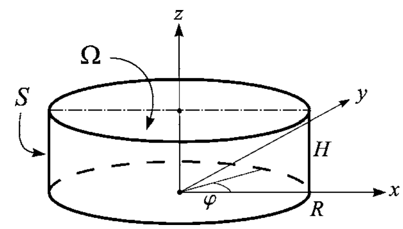

FIG. 1. The geometry of the cylindrical quasioptical cavity resonator. S is the resonator side face, Ω is its volume, H is the height of the cylinder, R is the radius of its base.

The possible applications of spectral study of the resonator with random inhomogeneities to nanoelectron systems are considered in the paper. Such a resonator can be a model of semiconductor quantum billiard. Based on our results we suggest the use of such billiards with spectrum rarefied by random inhomogeneities as an active system of semiconductor laser.

II. STATISTICAL SPECTRAL THEORY OF THE RESONATOR WITH RANDOM BULK INHOMOGENEITIES

A. Statement of the problem

Consider a cylindrical quasioptical cavity resonator of radius R and height H (see Fig. 1). The inner volume of the resonator Ω is assumed to be filled with the material having random inhomogeneous permittivity. We are interested in oscillations that constitute the transverse-electrical resonance mode (TE mode) provided that the resonator is empty. The vertical (z) component of the electrical field of this mode is equal to zero.

According to Ref. [13], the electromagnetic field of the TE mode can be calculated using the magnetic Hertz vector that has only one non-zero component, namely, z component $\Pi_z(\mathbf{r})$. We assume that inhomogeneity of the permittivity of the resonator infill is small. In this case, to define $\Pi_z(\mathbf{r})$ in the inhomogeneous resonator we use the approximate wave equation

$$[\Delta + k^2\varepsilon(\mathbf{r})]\Pi_z(\mathbf{r}) = 0, \tag{1}$$

where all components of the Hertz vector, except z component, are assumed to have a zero value provided that the inhomogeneity is adequately small. In Eq. (1), Δ is the three-dimensional (3D) Laplace operator, $\varepsilon(\mathbf{r})=\varepsilon_0+\delta\varepsilon(\mathbf{r})+i\alpha$ is the complex permittivity whose imaginary part α phenomenologically takes into account the ohmic loss in the system; the function $\delta\varepsilon(\mathbf{r})$ describes random spacial fluctuations of the permittivity around its average value ε_0, $k=\omega/c$ is the wave number.

In the case of a classical resonance system, the excitation by a given point monochromatic source is governed by Eq. (1) which should be complemented with δ term in the right-hand side. The equation thus obtained coincides in form with the equation for the Green function of quantum particles moving in a dissipative medium and being subjected to an inhomogeneous scalar potential. Therefore the results of the present formally electromagnetic study can be extended, at least qualitatively, to solid-state objects such as randomly inhomogeneous semiconductor quantum billiards of near-cylindrical shape.

In the presence of some dissipation mechanisms (say, the ohmic loss in the resonator walls) the equation for the Green function of Eq. (1) takes the form

$$[\Delta + k^2 - i/\tau_d - V(\mathbf{r})]G(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \tag{2}$$

Here τ_d is the dissipative attenuation time whose inverse value is connected to the imaginary part of function $\varepsilon(\mathbf{r})$ taken with minus sign. The potential $V(\mathbf{r})$ in the case of electromagnetic system is given by $V(\mathbf{r})=-k^2\delta\varepsilon(\mathbf{r})$. Boundary conditions for the solution to Eq. (2) result from the requirement of vanishing the tangential components of electrical field of the TE mode at the resonator interface. On the side boundary S, the Neuman condition

$$\left.\frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial r}\right|_S = 0 \tag{3a}$$

should be met whereas at end surfaces $z=\pm H/2$ the Dirichlet condition

$$G(\mathbf{r},\mathbf{r}')|_{z=\pm H/2} = 0 \tag{3b}$$

should be satisfied.

To study the oscillation spectrum of the resonator with random inhomogeneities, the poles of the Green function averaged over realizations of the potential $V(\mathbf{r})$ from Eq. (2) should be determined. For example, this function can be found from the Dyson equation. Yet by now the efficient methods for solving this equation in the case of confined multidimensional systems do not exist. For this purpose we apply the original calculation technique which relies on precise separation of quantization modes in an arbitrary confined system, including the disordered one. The mode separation method had been previously developed to solve transport problems in disordered 2D open systems [11]. Then it was modified for three dimensional systems of waveguide geometry in Refs. [12,14,15]. Below we extend this technique to the systems of closed geometry, in particular, to cavity resonators.

At the first step let us get on to mode representation of Eq. (2) using a set of orthonormal basis functions. The whole set of eigenfunctions of the Laplace operator appears to be the most suitable for our purpose. For the case of the cylindrical resonator shown in Fig. 1 these functions can be given to the form

$$|\mathbf{r},\mu\rangle = |r,\varphi;l,n\rangle|z,q\rangle, \tag{4}$$

where $\mathbf{r}=(r,\varphi,z)$ is the radius vector in cylindrical coordinates, $\mu=(l,n,q)$ is the vectorial mode index conjugate to that vector. Normalized eigenfunctions of the "transverse" part of the Laplacian, which obey boundary conditions Eq. (3), are given by

$$|r,\varphi;l,n\rangle = C_{ln}/(\sqrt{\pi}R)J_{|n|}(\gamma_l^{(|n|)}r/R)e^{in\varphi}, \tag{5a}$$

$$l=1,2,\ldots, \quad n=0,\pm 1,\pm 2,\ldots,$$

where the coefficient C_{ln} has the form

$$C_{ln} = \frac{\gamma_l^{(|n|)}}{[(\gamma_l^{(|n|)})^2-n^2]^{1/2}J_{|n|}(\gamma_l^{(|n|)})}. \tag{5b}$$

The set of coefficients $\gamma_l^{(|n|)}$ in Eqs. (5) consists of positive zeros of the function $J'_{|n|}(t)$ which are numbered by index l in ascending order. The eigenvalues corresponding to functions Eq. (5a) are equal to $\xi_{ln}=-(\gamma_l^{(|n|)}R)^2$. Basis functions of the "longitudinal" part of the Laplacian $\partial^2\partial z^2$, which meet boundary conditions (3b), are given by

$$|z;q\rangle = \sqrt{\frac{2}{H}}\sin\left[\left(\frac{z}{H}+\frac{1}{2}\right)\pi q\right],$$

$$q=1,2,\ldots, \tag{6}$$

the corresponding eigenvalues being equal to $(\pi q/H)^2$.

In the basis of functions (4), Eq. (2) takes the form

$$(k^2-\kappa_\mu^2-i/\tau_d-\mathcal{V}_\mu)G_{\mu\mu'} - \sum_{\nu\neq\mu}\mathcal{U}_{\mu\nu}G_{\nu\mu'} = \delta_{\mu\mu'}. \tag{7}$$

Here, $G_{\mu\mu'}$, is the Green function in mode representation, the parameter

$$\kappa_\mu^2 = \left(\frac{\gamma_l^{(|n|)}}{R}\right)^2 + \left(\frac{\pi q}{H}\right)^2 \tag{8}$$

is the unperturbed "energy" of the mode μ (the eigenvalue of 3D Laplace operator), functions $\mathcal{U}_{\mu\nu}$ are mode matrix elements of the random potential, viz.

$$\mathcal{U}_{\mu\nu} = \int_\Omega d\mathbf{r}\langle\mathbf{r};\mu|V(\mathbf{r})|\mathbf{r};\nu\rangle. \tag{9}$$

Particular attention should be paid to the fact that the intramode, i.e. diagonal in mode indices, matrix element $\mathcal{U}_{\mu\mu}\equiv\mathcal{V}_\mu$ is separated in Eq. (7) from other terms of the sum where only matrix elements corresponding to intermode scattering are thus kept. It was shown in Ref. [11] that such a separation of intramode and intermode effective potentials provides mathematical correctness of the derivation of closed equations for the diagonal components of Green matrix $\|G_{\mu\mu'}\|$ and with those components for the entire matrix integrally.

B. Separation of the modes

The solution of infinite set of coupled equations (7) is no less an intricate problem than a direct solution of multidimensional differential equation (2). The problem would be resolved in a straightforward fashion on condition that the resonator modes allow their strict separation. Normally, the modes are easily separable if the resonator is quite symmetrical, and has no random inhomogeneities. Below, based on the techniques developed in Refs. [11,12,14], it will be shown that in the case of simple-shaped (integrable) unperturbed resonator, the modes can be strictly separated even if there is an arbitrary bulk inhomogeneity. But, in general, the cost of this separation is the appearance of the effective potentials in equations for each of the modes. These potentials are known as T matrices in quantum theory of scattering [16], their functional structure being much more involved than that of the initial potential $V(\mathbf{r})$.

As a starting point for mode separation we introduce unperturbed (or trial) mode propagator $G_\nu^{(V)}$ by omitting in Eq. (7) all intermode potentials $\mathcal{U}_{\mu\nu}$,

$$G_\nu^{(V)} = (k^2 - \kappa_\nu^2 - i/\tau_d - \mathcal{V}_\nu)^{-1}. \tag{10}$$

The term "unperturbed" will be used hereupon with respect to intermode potentials, intramode ones being taken into account rigorously.

By substituting $\mu' = \mu$ in Eq. (7) we obtain a linear nonuniform connection of intramode propagator $G_{\mu\mu}$ with all intermode Green functions having the particular right-hand mode index μ,

$$G_{\mu\mu} = G_\mu^{(V)}\left(1 + \sum_{\nu\neq\mu} \mathcal{U}_{\mu\nu} G_{\nu\mu}\right). \tag{11}$$

Assuming then $\mu' \neq \mu$ and performing some necessary relabeling of mode indices we can reduce Eq. (7) to the form

$$[G_\nu^{(V)}]^{-1} G_{\nu\mu} - \sum_{\substack{\nu'\neq\nu\\ \nu'\neq\mu}} \mathcal{U}_{\nu\nu'} G_{\nu'\mu} = \mathcal{U}_{\nu\mu} G_{\mu\mu} \quad (\nu \neq \mu). \tag{12}$$

The latter system of interconnected linear equations can be readily solved with respect to intermode elements of the Green matrix. In the operator form the solution is given by

$$G_{\nu\mu} = \hat{\mathbf{P}}_\nu (1 - \hat{R})^{-1} \hat{R} \hat{\mathbf{P}}_\mu G_{\mu\mu}, \tag{13}$$

where the linear operator $\hat{R} = \hat{G}^{(V)} \hat{\mathcal{U}}$ of intermode scattering is introduced. This operator acts in the mode subspace $\bar{M}_\mu$ consisting of the whole set of mode indices but the index μ; $\hat{\mathbf{P}}_\mu$ is the projection operator whose action reduces to the assignment of the value μ to the nearest mode index of any adjacent operator, no matter where it may stand—to the left or to the right of $\hat{\mathbf{P}}_\mu$. Operators $\hat{G}^{(V)}$ and $\hat{\mathcal{U}}$ are specified on $\bar{M}_\mu$ by matrix elements

$$\langle \nu | \hat{G}^{(V)} | \nu' \rangle = G_\nu^{(V)} \delta_{\nu\nu'}, \tag{14a}$$

$$\langle \nu | \hat{\mathcal{U}} | \nu' \rangle = \mathcal{U}_{\nu\nu'}. \tag{14b}$$

Correspondingly, the matrix elements of the operator $\hat{R}$ are given by

$$\langle \nu | \hat{R} | \nu' \rangle = G_\nu^{(V)} \mathcal{U}_{\nu\nu'}. \tag{15}$$

By substituting intermode propagators in the form (13) into the relationship (11), we obtain the following rigorous expression for intramode propagator $G_{\mu\mu}$:

$$G_{\mu\mu} = (k^2 - \kappa_\mu^2 - i/\tau_d - \mathcal{V}_\mu - \mathcal{T}_\mu)^{-1}. \tag{16}$$

Here

$$\mathcal{T}_\mu = \hat{\mathbf{P}}_\mu \hat{U} (1 - \hat{R})^{-1} \hat{R} \hat{\mathbf{P}}_\mu \tag{17}$$

is the portion of the mode μ eigenenergy which is related to the intermode scattering.

It should be noted that in order to determine the disordered resonator spectrum it would suffice to find the poles of solely diagonal elements of the Green matrix. It is precisely these elements that determine all major analytical properties of the whole Green function of Eq. (2). In what follows we will analyze the cavity resonator spectrum with the use of the relatively simple statistical model of random potential $V(\mathbf{r})$.

C. Statistical analysis of the resonator spectrum

We suppose that the potential $V(\mathbf{r})$ has a zero mean value $\langle V(\mathbf{r}) = 0 \rangle$, and binary correlation function

$$\langle V(\mathbf{r}) V(\mathbf{r}') \rangle = D W(\mathbf{r} - \mathbf{r}'). \tag{18}$$

Considering the forthcoming numerical analysis we will take the function $W(\mathbf{r})$ in the form of Gaussian exponent, viz. $W(\mathbf{r}) = \exp(-\mathbf{r}^2/2r_c^2)$, where r_c stands for the correlation radius. In the case of electromagnetic resonator the normalization constant D in Eq. (18) is given by $D = k^4\sigma^2$, where $\sigma^2 = \langle \delta\varepsilon^2(\mathbf{r}) \rangle$ is the variance of permittivity fluctuations.

The pair of selected statistical parameters, i.e., the average random potential and its binary correlation function, are sufficient to make a detailed analysis of the system in study if function $\delta\varepsilon(\mathbf{r})$ is the Gaussian-distributed random variable. Yet these two parameters are also sufficient for doing an asymptotically correct analysis even in the case where statistics of the fluctuations is markedly non-Gaussian, provided that the potential $V(\mathbf{r})$ is, in a way, a small one. As is customary in condensed matter physics, we will regard the potential to be small and the resulting scattering to be weak if the scattering rate calculated in Born approximation is small as compared to the unperturbed quasiparticle energy (k^2 in our particular case). The smallness of the onefold scattering probability enables one to regard the potential $V(\mathbf{r})$ with a parametric accuracy as a Gaussian random process, whatever its distribution [17].

Now consider the self-energy operator of the mode μ which consists, in accord with Eq. (16), of two terms

$$\Sigma_\mu = V_\mu + \mathcal{T}_\mu. \tag{19}$$

The first term in the right-hand side (RHS) of this formula vanishes when being averaged whereas the second one does not. Its average value can be easily calculated under the assumption of weak scattering. The strength of the intermode scattering is estimated by the norm of the operator $\hat{R}$ entering Eq. (17). Assuming this norm to be small as compared to unity and keeping only two main terms in the expansion of the inverse operator in Eq. (17) we find it necessary to average not the exact operator potential T_μ but rather its relatively compact limiting value

$$\mathcal{T}_\mu \approx \hat{\mathbf{P}}_\mu \hat{U} \hat{G}^{(V)} \hat{\mathcal{U}} \hat{\mathbf{P}}_\mu = \sum_{\nu\neq\mu} \mathcal{U}_{\mu\nu} G_\nu^{(V)} \mathcal{U}_{\nu\mu}. \tag{20}$$

When calculating the quantity $\langle \mathcal{T}_\mu \rangle$ one can neglect, with a parametric accuracy, the correlation between intramode and intermode potentials. This permits us to average intramode propagator $G_\nu^{(V)}$ in Eq. (20) and its envelopes consisting of intermode potentials $\mathcal{U}_{\mu\nu}$ independently. To average the function $G_\nu^{(V)}$ it is worthwhile to present it in the integral form

$$G_\nu^{(V)} = \int_0^\infty dt \exp[-i(k^2 - \kappa_\nu^2 - i/\tau_d - V_\nu)t]. \tag{21}$$

Then averaging of the integrand in Eq. (21) by means of the continual integration with Gaussian functional weight yields

$$\begin{aligned}\langle G_\nu^{(V)} \rangle &= \frac{1}{k^2}\int_0^\infty dt \exp\left[-i(1-\kappa_\nu^2/k^2 - i/k^2\tau_d)t - \frac{t^2}{2}\sigma^2 L_\nu(r_c)\right] \\ &= \frac{1}{k^2}\sqrt{\frac{\pi}{2\sigma^2 L_\nu(r_c)}} \exp\left[-\frac{(k^2-\kappa_\nu^2 - i/\tau_d)^2}{2k^4\sigma^2 L_\nu(r_c)}\right] \\ &\quad\times\left\{1 - \Phi\left[\frac{i(k^2-\kappa_\nu^2-i/\tau_d)}{\sqrt{2k^4\sigma^2 L_\nu(r_c)}}\right]\right\}. \end{aligned} \tag{22}$$

Here $\Phi(\xi)$ is the probability integral [18], $L_\nu(r_c)$ is the dimensionless autocorrelator of intramode potential $\mathcal{V}_\nu$. In the case of Gaussian correlation function $W(\mathbf{r})$ it is written as

$$\begin{aligned} L_\nu(r_c) &= \frac{1}{k^4}\langle \mathcal{V}_\nu \mathcal{V}_\nu \rangle = \int\int_\Omega d\mathbf{r} d\mathbf{r}' \langle \mathbf{r},\nu|\mathbf{r},\nu\rangle \exp[-(\mathbf{r}-\mathbf{r}')^2/2r_c^2] \\ &\quad\times\langle \mathbf{r}',\nu|\mathbf{r}',\nu\rangle = \frac{8}{\pi} C^4_{l_\nu n_\nu} \int_0^1\int_0^1 ds ds' \\ &\quad\times\exp\left[-\frac{H^2}{2r_c^2}(s-s')^2\right]\sin^2(\pi q_{n_\nu} s)\sin^2(\pi q_{n_\nu} s') \\ &\quad\times\int_0^1\int_0^1 tt' dt dt' J^2_{|n_\nu|}(\gamma_{l_\nu}^{|n_\nu|} t) J^2_{|n_\nu|}(\gamma_{l_\nu}^{|n_\nu|} t') \oint d\varphi \\ &\quad\times\exp\left[-\frac{R^2}{2r_c^2}(t^2 + t'^2 - 2tt'\cos\varphi)\right]. \end{aligned} \tag{23}$$

Although expression (22) is formally exact, it is not quite convenient for making further analysis in view of its bulky structure. Asymptotical calculations at large and small values of the probability integral argument permit us to use a far simpler interpolation expression for $\langle G_\nu^{(V)} \rangle$, namely,

$$\langle G_\nu^{(V)} \rangle \approx (k^2 - \kappa_\nu^2 - i/\tau_\nu^*)^{-1}, \tag{24a}$$

which is close in form to the initial unperturbed Green function (10). Here we use the notation

$$1/\tau_\nu^* = 1/\tau_d + 1/\tau_\nu^{(\phi)} \tag{24b}$$

for the effective scattering frequency. The latter includes both the initial dissipative term $1/\tau_d$ and the addendum $1/\tau_\nu^{(\phi)} = k^2\sqrt{(2/\pi)\sigma^2 L_\nu(r_c)}$ originating from wave scattering by random non-dissipative inhomogeneities present in the bulk of the resonator.

The second term in the RHS of Eq. (24b) is responsible for the effect analogous to that produced by the first, dissipative term. Specifically, it results in additional widening of the mode ν trial resonance line whose shape is determined by the modulus of the function (24a). However, in contrast to the dissipative widening the effect produced by the term $1/\tau_\nu^{(\phi)}$ has basically nothing to do with true dissipation. This additional width of the resonance originates entirely from nondissipative (elastic) scattering of excited harmonics by inhomogeneities randomly placed on the propagation path. Note that this type of widening survive even in systems where the energy loss goes to zero, i.e., $\tau_d \to \infty$.

The widening associated with the inhomogeneity of resonance systems (usually side boundary imperfections) is normally referred to as the nonuniform widening. In our case, where the resonator is randomly inhomogeneous in the bulk, its physical origin can be easily interpreted in terms of the rays of waves propagating within a closed area. It is well known that resonator modes can be thought of as being formed as a result of interference of waves propagating in opposite directions along closed trajectories. In reasonably symmetric cavities such trajectories constitute well-resolved quantized sets. The addition to the cavity of random scatterers, either surface or bulk, should break down the initially perfect interference picture. In the course of multiple scattering each of the initially closed wave trajectories in the cavity decomposes into a quite dense bundle of trajectories, whose effective width is determined by the particular scattering parameters. As a result, a set of compactly placed resonances replaces each of the initially well-resolved peaks thus allowing one to interpret this set as the widening of initial resonances.

That the origin of the term $1/\tau_\nu^{(\phi)}$ in Eq. (24b) is not related to dissipation but rather to phase correlation of waves randomly scattered in the resonator. It makes possible to interpret this scattering rate as the dephasing frequency. This physical concept is conventional in condensed matter physics [19]. The above analysis suggests that the dephasing of quantum states can equally originate from inelastic (dissipative) scattering and from scattering caused by static disorder.

To make a further comparison of theoretical results to experimental data we will further consider the specific limiting case where the following inequalities are fulfilled:

$$r_c \lesssim H \ll R. \tag{25}$$

These restrictions enable us to calculate the integrals in Eq. (23) asymptotically, thus obtaining the following estimate for parameter $L_\nu(r_c)$: $L_\nu(r_c) \sim (r_c R)^2$. The correlator of intermode potentials in Eq. (20) can be written as

$$\langle U_{\mu\nu} U_{\nu\mu} \rangle = k^4 \sigma^2 \left(\frac{2}{\pi}\right)^2 \int_0^1 \int_0^1 ds ds' \exp\left[-\frac{H^2 (s-s')^2}{2r_c^2}\right] \times \sin(\pi q_{n_\mu} s) \sin(\pi q_{n_\mu} s') \sin(\pi q_{n_\nu} s) \sin(\pi q_{n_\nu} s') \times \int_0^1 \int_0^1 t t' dt dt' J_{|n_\mu|}(\gamma_{l_\mu}^{(|n_\mu|)} t) J_{|n_\mu|}(\gamma_{l_\mu}^{(|n_\mu|)} t') J_{|n_\nu|} \times (\gamma_{l_\nu}^{(|n_\mu|)} t) J_{|n_\nu|}(\gamma_{l_\nu}^{(|n_\mu|)} t') \oint \oint d\varphi d\varphi' \times \exp\left\{ -i(n_\mu - n_\nu)(\varphi - \varphi') - \frac{R^2}{2r_c^2}[t^2 + t'^2 - 2tt' \cos(\varphi - \varphi')] \right\}. \quad (26)$$

Asymptotic calculations of the integrals over φ and φ' yield the following expression:

$$\langle \mathcal{U}_{\mu\nu} U_{\nu\mu} \rangle = k^4 \sigma^2 \frac{r_c}{R} A_{\mu\nu}(r_c), \quad (27)$$

where factor $A_{\mu\nu}(r_c)$ is given by a rather cumbersome integral, viz.

$$A_{\mu\nu}(r_c) = \frac{8}{\sqrt{\pi}} C_{l_\mu n_\mu}^2 C_{l_\nu n_\nu}^2 \int_0^1 \int_0^1 ds ds' \exp\left[-\frac{H^2}{2r_c^2}(s-s')^2\right] \times \sin(\pi q_{n_\mu} s) \sin(\pi q_{n_\mu} s') \sin(\pi q_{n_\nu} s) \sin(\pi q_{n_\nu} s') \times \int_0^1 \int_0^1 \sqrt{tt'} dt dt' \exp\left[-\frac{R^2}{2r_c^2}(t-t')^2 - \frac{r_c^2 (n_\mu - n_\nu)^2}{4R^2 tt'}\right] J_{|n_\mu|}(\gamma_{l_\mu}^{(|n_\mu|)} t) J_{|n_\mu|}(\gamma_{l_\mu}^{(|n_\mu|)} t') J_{|n_\nu|} \times (\gamma_{l_\nu}^{(|n_\nu|)} t) J_{|n_\nu|}(\gamma_{l_\nu}^{(|n_\nu|)} t'). \quad (28)$$

Given the results (24) and (27), separation of real and imaginary parts of the average T-matrix, $\langle \mathcal{T}_\mu \rangle = \Delta k_\mu^2 + i/\tau_\mu^{(ch)}$, leads to the following expressions describing the shift and the broadening of the μth resonant level:

$$\Delta k_\mu^2 = k^4 \sigma^2 \frac{r_c}{R} \sum_{\nu \neq \mu} A_{\mu\nu}(r_c) \mathrm{Re} \langle G_\nu^{(V)} \rangle, \quad (29a)$$

$$\frac{1}{\tau_\mu^{(ch)}} = k^4 \sigma^2 \frac{r_c}{R} \sum_{\nu \neq \mu} A_{\mu\nu}(r_c) \mathrm{Im} \langle G_\nu^{(V)} \rangle. \quad (29b)$$

The factor $A_{\mu\nu}(r_c)$, as one can ascertain from Eq. (28), is a real-valued quantity, its absolute value being estimated by parameter $r_c/R \ll 1$. Although the sign of this factor cannot be uniquely identified in the general case, since it contains the dependence on specific indices of modes between which the scattering is carried out, the numerical analysis shows that $A_{\mu\nu}(r_c) > 0$. This inequality corresponds to the spectral lines broadening. One can adequately estimate both the shift and the broadening of the μ-th resonant level by substituting function $\langle G_\nu^{(V)} \rangle$ in the interpolated form (24a), instead of exact expression (22), into the right-hand sides of formulas (29). The result obtained reduces to

$$\Delta \kappa_\mu^2 = k^4 \sigma^2 \frac{r_c}{R} \sum_{\nu \neq \mu} A_{\mu\nu}(r_c) \frac{\kappa_\mu^2 - \kappa_\nu^2}{(\kappa_\mu^2 - \kappa_\nu^2)^2 + (1/\tau_\nu^*)^2}, \quad (30a)$$

$$\frac{1}{\tau_\mu^{(ch)}} = k^4 \sigma^2 \frac{r_c}{R} \sum_{\nu \neq \mu} A_{\mu\nu}(r_c) \frac{1 \tau_\nu^*}{(\kappa_\mu^2 - \kappa_\nu^2)^2 + (1/\tau_\nu^*)^2}. \quad (30b)$$

The structure of the summands in Eqs. (30) indicates unambiguously that both the shift and the broadening of each given resonance ($k^2 \simeq k_\mu^2$) are mainly provided by its interactions with the adjacent resonances, whose position on the frequency scale are confined to the region limited by order equality $|\kappa_\mu^2 - \kappa_\nu^2| \sim 1/\tau_\nu^*$. In the case where only one resonance level with, say, frequency $\bar{\omega}_\nu$ proves falls into the above indicated interval around the μth resonance (we assume that the condition $\sigma r_c/R \ll 1$ of weak intramode scattering is satisfied), its contribution to the shift and the width of the μth resonance is estimated as

$$\omega_\mu - \omega_{\mu 0} = \frac{\sigma^2 \lambda A_{\mu\nu}(\omega_{\mu 0} - \bar{\omega}_\nu)}{(\delta\omega_{\mu 0} + \sigma\lambda)^2}, \quad (31a)$$

$$\delta\omega_\mu = \frac{\sigma^2 \lambda A_{\mu\nu}}{\delta\omega_{\mu 0} + \sigma\lambda}. \quad (31b)$$

Here $\omega_{\mu 0}$ and ω_μ are the cyclic spectral frequency of empty resonator and that of the resonator filled with random inhomogeneities, respectively, $\delta\omega_{\mu 0}$ and $\delta\omega_\mu$ are the relative widths of their spectral lines, the parameter $\lambda \sim r_c/R$. If several resonances fall into the indicated vicinity of the μth resonance simultaneously, they contribute additively to both the level position and and the width.

From the above theory it follows that filling of a quasi-optical cavity resonator with randomly distributed inhomogeneities results both in the random shift of its spectral lines and an increase of their line widths. This type of widening is not related to dissipation in the cavity and may be interpreted as nonuniform widening. Here the theory can give only qualitative predictions for experiment. This situation is quite similar to the wave scattering by randomly rough surfaces [2]. At the same time the problem of oscillations in the quasioptical cavity resonator filled with bulk random inhomogeneities differs essentially from that of wave scattering by a randomly rough surface.

The point is that oscillations in the cavity resonator are formed due to multiple transmissions of waves (with multiplicity of the order of Q) through its random infill. Bearing this in mind one might expect that with the condition $r_c \ll R$ the characteristics of spectral lines are effectively self-averaged [17]. In this case the agreement between theory and experiment may be achieved at small number of measurements or even for one particular realization of the random system. The confirmation or the denial of this assumption may be achieved by comparing theoretical predictions to the experimental data.

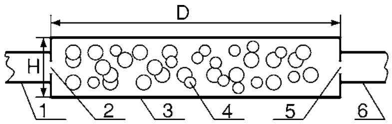

FIG. 2. The quasioptical cylinder cavity millimeter wave resonator filled with inhomogeneities, 1 and 6 are input/output waveguides, 2 and 5 are the holes coupling the resonator with waveguides, 3 is the resonator body, 4 are the styrofoam particles, D is the resonator diameter, H is its height; $D=130$ mm, $H=14$ mm.

III. EXPERIMENT AND DISCUSSION

The main goal of our experiment is to verify the theoretical results for shifting and broadening of spectral lines caused by inhomogeneities and the possible resonator spectrum "rarefaction." Yet another goal is as follows. In the paper [7] we have detected strong spectrum stochastization caused by inhomogeneities such as anisotropic sapphire particles having the dimensions of order of an operating wavelength, which are placed into the cavity resonator. The spectrum is mixed due to both a regular and a chaotic parts of the spectrum. In contrast to Ref. [7], in the present paper we examine the influence on the resonator spectrum of relatively small isotropic inhomogeneities. Such inhomogeneities can be made from styrofoam particles with an average permittivity value close to unity and with a small dielectric loss angle. Thus, the following question arises:Can the spectrum chaotization be made possible in this case?

A. Experiment technique

We look into the influence of inhomogeneities on the cavity resonator spectrum at a frequency range of 32–37 GHz. A quasioptical cylinder millimeter wave resonator random filled with styrofoam particles (Fig. 2) is chosen for the experiment. The styrofoam particles have the real part of permittivity of about unity and with small dielectric loss $\varepsilon=1.04+i10^{-4}$.

To study the influence of dielectric particles on the resonator spectrum it is necessary to provide a high quality factor for the oscillations in the inhomogeneity-free resonator. For this purpose we excited the TE mode in the resonator. The magnetic field vector of this mode is directed along the resonator z axis, and microwave currents do not cross the interface between the flat resonator face and cylinder surface. Owing to this the empty resonator has a high quality factor up to 2×10^4. To excite the selected mode we used a waveguide diffraction antenna. It represents the circular hole with a diameter of 2 mm in a thin diaphragm 0.1 mm thick, which closes the input waveguide. The diaphragm surface is flush mounted with the side-cut resonator cylinder surface. The identical antenna is used to receive the oscillations on the opposite side of the cylinder surface.

The resonator spectrum was detected using "on pass" regime over 32–37 GHz frequency range by a wide-band standing wave ratio meter. The measurement process was computerized. A signal from the measurement device goes into computer using analog-digital conversion. The further signal processing (determining the spectral line intensity, its quality factor, and frequency) was performed by applying the special software package. This makes it possible to handle the measurement data for a huge number of realizations of the random inhomogeneities in a short period of time. Owing to this, the the accuracy of frequency and quality factor measurement do not exceed 0.1 and 1 %, respectively. The styrofoam particles used as inhomogeneities are of approximately 2–3 mm size. Their space distribution among them was arbitrary for each realization. The spectral characteristics were measured depending on the number of these inhomogeneities.

B. Statistical analysis of the resonator spectrum

As the number of inhomogeneities increases, the spectrum of the quasioptical resonator assumes a stochastic nature that is visualized in the distribution of IF intervals. In order to define the relation between regular and random spectral components, the comparison of IF interval distribution obtained experimentally to different theoretical distributions based on *a priori* data on a statistical process is commonly used. Specifically, to find the distribution of the IF interval probability we use the Brody function $P_B(s)$ given by

$$P_B(s) = As^{\beta}\exp(-Bs^{1+\beta}), \quad (32)$$

where $s=(\omega_n-\omega_{n-1})\rho(\omega_n)$, ω_n is the spectral line frequency, $\rho(\omega_n)$ is the spectral density, i.e., the superposition of regular and random motion density, β is the measure of stochastic motion, constants A and B are defined from the standardization condition $A=(1+\beta)B$, $B=\Gamma^{1+\beta}(2+\beta)(1+\beta)^{-1}$, $\Gamma(z)$ is the gamma function. At $\beta\to0$ the IF intervals in the spectrum are not correlated and can be described by the Poisson distribution, and at $\beta\to1$ we have the Wigner distribution, in which the repulsion effect of spectral lines exists. In other words, the probability of closest-to-zero IF interval is equal to zero.

If the measure of the spectrum stochastization is relatively small and the distribution of IF intervals is close to the Poissonian one we can use the Berry-Robnik distribution $P_{\mathrm{BR}}(s)$ [20] expressed as

$$P_{\mathrm{BR}}(s) = \rho^2 e^{-\rho s}\,\mathrm{erfc}\left[\frac{\sqrt{\pi}}{2}(1-\rho)s\right] + \left[\frac{\pi}{2}(1-\rho)^2 s + 2\rho\right] \times(1-\rho)e^{-\rho s-(\pi/4)(1-\rho)^2 s^2}, \quad (33)$$

where $\mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-t^2)dt$, ρ is the relative phase volume taken up by a regular trajectory in a mixed system. The limit $\rho\to1$ corresponds to the regular system; $\rho\to0$ corresponds to a completely chaotic one. The value $1-\rho$ is relative phase volume occupied by chaotic motion.

The experimental data indicate that in the empty resonator the IF intervals distribution is sufficiently close to the Berry-Robnik distribution with $\rho=1$ [the Poisson distribution $P(s)\sim\exp(-s)$] [Fig. 8(a)]. By increasing the number of inhomogeneities (styrofoam particles) the function $P(s)$ has a maximum value at small s. The presence of the maximum in $P(s)$ is indicative of the spectral line repulsion effect. The

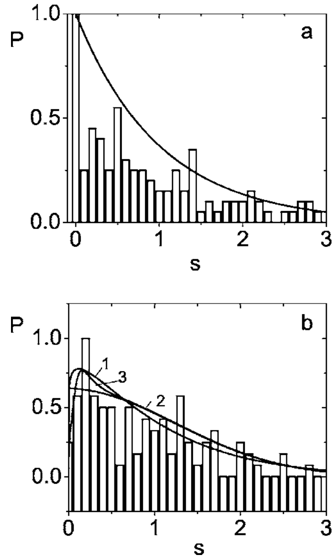

FIG. 3. The nearest-neighbor spacing distribution, $P(s)$. (a) is for the empty resonator spectrum. The solid line is for the Berry-Robnik distribution at $\rho \to 1$ or the Brody distribution at $\beta \to 0$. (b) is for the spectrum of the resonator entirely filled with styrofoam particles. Curve 1 is for the Brody distribution at $\beta = 0.1$; curve 2 is for the Berry-Robnik distribution at $\rho = 0.4$; curve 3 is for the Podolskiy-Narimanov distribution at $\rho = 0.4$ and $\nu = 0.1$ [29].

random component thus increases. We have found out that with a resonator being completely filled with styrofoam particles we have $\rho = 0.4$ [Fig. 3(b)].

As evident from Fig. 4 the function $P(s)$ has the maximum at small s which is not described by the Berry-Robnik dependence [21]. This maximum can be explained by chaos-assisted tunneling (CAT) effect [22] in the resonator filled with random inhomogeneities. The IF distribution function proposed in [22] describes the observed distribution for parameter $\rho = 0.4$, which corresponds to classical dynamics, and for parameter $\nu = 0.1$ describing the tunneling between different modes.

We calculated the spectral rigidity for the resonator filled with inhomogeneities. The spectral rigidity $\Delta_3(L)$ is an integral characteristic of the degree of spectral lines ordering for frequency distances that are much longer than the IF interval. It is given by [23]

$$\Delta_3(x,L) = \frac{1}{L} \min_{A,B} \int_x^{x+L} [n(\varepsilon) - A\varepsilon - B]^2 d\varepsilon, \qquad (34)$$

where L is the interval where the function $\Delta_3(x,L)$ is determined. The function $n(\varepsilon)$ is constructed as follows [23]. For a sequence of frequencies ω_n normalized to unit density ($\omega_n = \omega_{n-1} + S_n$) we introduce a staircase function $n(\varepsilon)$ equal to the number of frequencies lying below ε.

The function $n(\varepsilon)$ has a staircase form with average unit tilt. The function $\Delta_3(x,L)$ is defined as the minimum of quadratic deviation of $n(\varepsilon)$ from the straight line in the interval $(x, x+L)$. The meaning of spectral rigidity (34) averaged over x, $\langle \Delta_3(x,L), \rangle_x$, depends on L only and is denoted as $\Delta_3(L)$.

The curve $\Delta_3(L)$ for the resonator with random inhomogeneities is shown in Fig. 4. This curve marked as (2) is placed between the spectral rigidity for the Poisson distribution (curve 1, $[\Delta_3(L) = L/15]$) and curve 3 which corresponds to the spectral rigidity for the Gaussian orthogonal ensemble (GOE), $\Delta_3(L) = 1/\pi^2 \ln L - 0.00687$ [23]. The spectral rigidity related to GOE is observed when modeling the quantum chaos in unstable microwave cavity resonators similar to Sinai and Bunimovich billiards [24].

IV. SHIFTING AND BROADENING OF SPECTRAL LINES: EFFECT OF SPECTRUM "RAREFACTION"

To compare the theory and the experiment we calculate the spectrum of TE modes in the quasioptical cylinder cavity millimeter wave resonator filled with random dielectric inhomogeneities using our theoretical results. When calculating the resonator spectrum the resonator parameters were taken from the experiment. The spectrum was found by solving the excitation problem with a point external dipole. Figures 5(a)–5(d) shows the influence of random bulk inhomogeneities on the spectrum at different values of parameter σ calculated with Eqs. (30). As an example we selected the spectrum at the 36–37 GHz frequency interval.

It is shown that the spectrum has considerably changed in the presence of bulk dielectric inhomogeneities. The magnitude of spectral lines shifting and broadening depends on the level of filling the resonator by inhomogeneities. The solitary spectral lines retain their high quality factor and their broadening is small enough [for example, the spectral line with $n = 15$, Figs. 5(a)–5(d)] even in the presence of inhomogeneities. At the same time the shift and the broadening of nearest-neighbor lines are significant. As one can see in Figs. 5(e) and 5(f), experimental results are in good agreement with our numerical data. Figures 5(g)–5(i) shows the theoretical amplitude-frequency dependence of the resonator spectrum constructed as a frequency dependence of Green function modulus at different values of parameter σ. These dependences suggest that the number of lines with a high quality factor substantially decreases if the number of inhomogeneities is becoming larger [Figs. 5(h) and 5(i)]. The reduction of the number of high quality lines can be explained as "rarefaction" of the resonator spectrum. The "rarefaction" is caused by dissimilar scattering conditions for different modes. So, as the number of inhomogeneities increases, the adjacent resonances tend to overlap and the quality factor of combined resonances becomes smaller (see, for example, spectral lines near 36.8 GHz).

Now examine the mechanism for spectral lines shift and broadening, which is dependent upon both the frequency distance between nearest-neighbor spectral lines and closeness of their azimuthal indexes. The interaction between the nearest-neighbor spectral lines becomes much stronger if these lines overlap and their azimuthal indexes are close. For example, the peaks near 36.8 GHz with $n_\mu = 2, 45, 0, 27, 21$ (36.7524, 36.7775, 36.7815, 36.795, 36.804 GHz) have both a short frequency distance and close azimuthal indexes ($n_\mu = 2, 0$) if the resonator is empty [Figs. 5(g) and 5(j)]. If the resonator is filled with inhomogeneities these peaks overlap and merge into almost one resonance with a rather low quality factor [Figs. 5(h), 5(i), and 5(k)]. It should be added that the peak with $n_\mu=2$ has the largest broadening and the negative relative frequency shift in the frequency band [Figs. 5(a)–5(f)] under consideration. This is due to the strong interaction between the nearest-neighbor peaks and the peaks that have $n_\mu=0$ and $n_\mu=4$. This claim can be proved as follows. Figure 6 shows the value of some terms from Eqs. (30a) and (30b). As one can see, the largest values correspond to the resonances with $n_\mu=0$ and $n_\mu=4$ for the dependence for $n_\mu=2$. It is necessary to note that for the solitary peak with $n_\mu=15$ all elements of the sum in Eq. (30) are small, therefore, their relative shift and broadening are small as well. The x dependence for the peak with $n_\mu=45$, which is not solitary and has some nearest-neighbor peaks, demonstrates the behavior similar to that of the peak with $n_\mu=15$ because these peaks do not have azimuthal indexes close to $n_\mu=45$. The amplitude of spectral lines is reduced [Figs. 5(h), 5(i), and 5(k)] and shift and broadening become depended upon both the frequency distance between nearest-neighbor spectral lines and on the closeness of their azimuthal indexes.

The theoretical and experimental spectrum of the empty resonator is rather dense and consists of 84 narrow spectral lines in the range of 32–37 GHz (Fig. 7). Each line can be identified according to mode indexes if the number of inhomogeneities is fairly small. We found out that oscillations

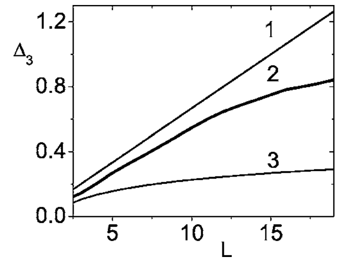

FIG. 4. The spectral rigidity for the resonator filled with styrofoam particles (curve 2). The straight line is for the spectral rigidity of the Poisson distribution (curve 1), curve 3 is for GOE.

with high quality factor (Q), of order of 10^4 and higher, have small azimuthal indexes ($n\sim1$) and high radial indexes ($l\gg1$). For the oscillation mode ($n\gg1, l=1$) the field is concentrated inside the resonator side-cut (whispering-gallery oscillations [9]), Q is nearly 2×10^3. The spectrum of the resonator with inhomogeneities differs essentially from the spectrum of empty resonator. By increasing the number of inhomogeneities the spectral lines are becoming wider, and Q, correspondingly, decreases. The intensity of broaden lines decreases too. The spectrum keeps only few lines with sufficiently high intensity and the Q value. For such lines the value of Q is close to the Q factor of spectral lines of the empty resonator. So, the number of lines with high intensities and with $Q>10^4$ is equal to 52 in the empty resonator [Fig.

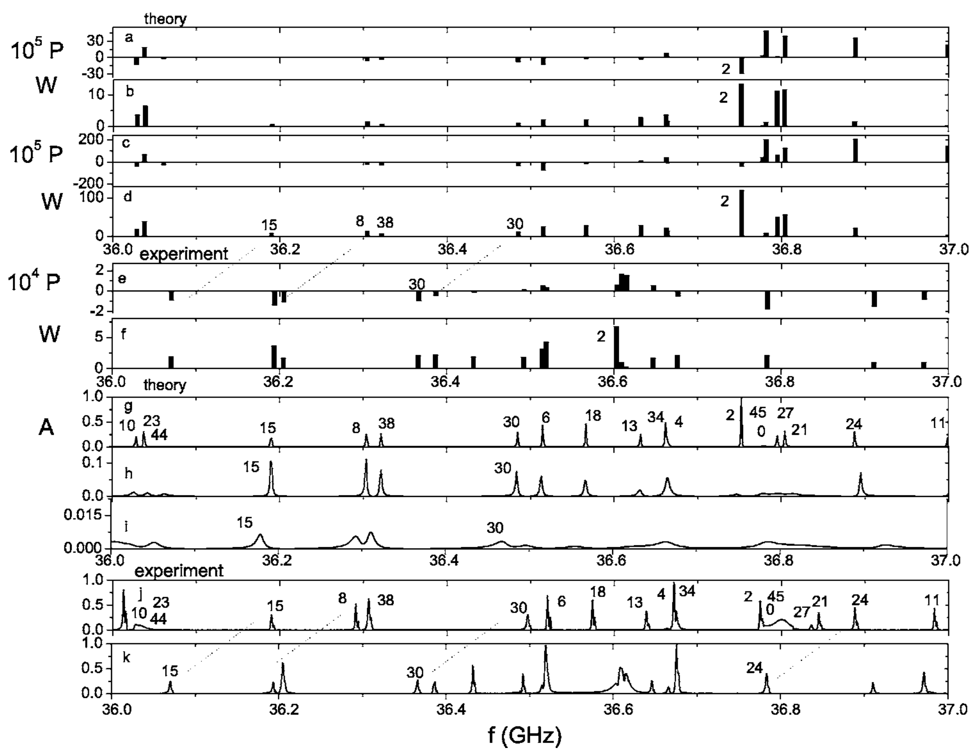

FIG. 5. Comparison of theoretical and experimental results. The dependence of relative spectral lines shift $P=\Delta\kappa_\mu^2/\kappa_\mu^2=2\Delta f_\mu/f_\mu$ and their relative broadening $W=\tau_d/\tau_\mu^{(\mathrm{ch})}=Q_d/Q_\mu$ on frequency f: (a)–(d) is the theory, and (e), (f) is the experiment. The theoretical amplitude-frequency dependence of the empty resonator spectrum (g) and the spectrum of the resonator with inhomogeneities (h),(i). The permittivity dispersion values of inhomogeneities are $\sigma=0.02$ (a),(b),(h) and $\sigma=0.05$ (c),(d),(i); $r_c=0.3$ cm. The experimental amplitude-frequency dependence of the empty resonator spectrum (j) and the spectrum of the resonator with inhomogeneities (k). The amplitude is normalized by maximal amplitude of resonances for the empty resonator in the considered wave range. The resonances are marked by numbers that are their azimuthal indexes. Dotted lines indicate spectral lines with the same azimuthal indexes.

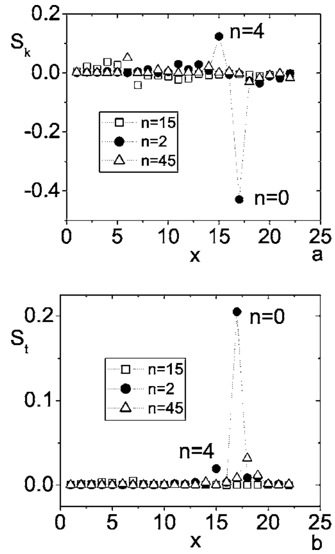

FIG. 6. The values of some terms in Eq. (30) for the resonances in the frequency band 38–37 GHz; n_μ=15, 2, 45 and different values of n_ν; (a) S_k is for the sum in Eq. (30a), (b) S_t is for the sum in Eq. (30b). The resonances are marked by their azimuthal indexes. The current numbers of resonances are put along the x axis.

7(a)]. At the resonator is filled by styrofoam particles the number of such lines is 10 [Fig. 7(b)], and at the resonator filled with pressed styrofoam particles, the number of lines is 3 [Fig. 7(c)].

Consider three highest peaks in Fig. 7(c). These peaks do not have so many nearest-neighbor peaks, as one can see in the insert in Fig. 7(c) with somewhat higher-resolution data. Their peak amplitudes are high as compared to other peaks because of the spectrum "rarefaction." In the inset of Fig. 7(c), there are many nearest-neighbor peaks with small amplitudes which are close to the highest peak due to the interaction between nearest resonances.

In the spectrum of inhomogeneous resonator there is a small number of lines with high Q value along with numerous low-intensity lines. This is equivalent to the spectrum

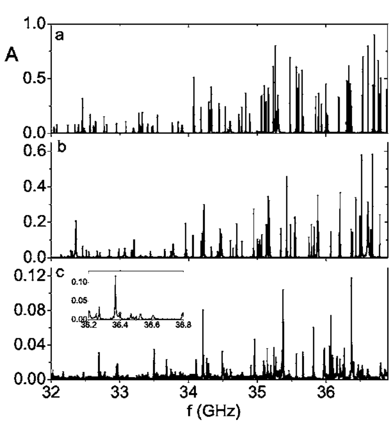

FIG. 7. The spectrum of the empty resonator (84 lines) (a); the resonator filled with styrofoam particles (77 lines) (b); the resonator filled with pressed styrofoam particles (57 lines) (c). The amplitude was normalized to the maximal amplitude value for the empty resonator.

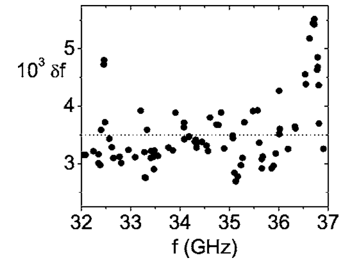

FIG. 8. The dependence of the frequency shift $\delta f=(f_{\text{empty}}-f_{\text{inhom}})/f_{\text{empty}}$ on f for the resonator filled with inhomogeneities. The dotted line is the value of the regular component of the shift.

"rarefaction" (Fig. 7). It has to be noted that the presence of high Q lines indicates that the broadening results, mainly, from the inhomogeneity of the resonator infill and is not specified by additional dissipation caused by small dielectric loss in the styrofoam. Along with the broadening of spectral lines they experience a frequency shift. This shift has both regular and random components (Fig. 8).

The regular shift occurs towards the low frequency range because of an increase in average dielectric permittivity of inhomogeneous medium in the resonator. It has an average value of 150 MHz for the resonator entirely filled with styrofoam particles.

It is important to note that if the distance between resonances is small enough they can either come closer or depart from each other because of the influence of inhomogeneities. Their Q factor is changed significantly (Figs. 9 and 10): one of them may increase whereas the other decreases.

Both the spectral lines broadening and their shift agree with described above theory and can be interpreted in the terms of intermode scattering. We can give simple physical understanding of the effect of broadening and shift of spectral lines in the quasioptical cylinder resonator filled with inhomogeneities. This interpretation is based on the ray treatment of resonant oscillations. Each of a resonant frequency corresponds to a periodical trajectory which optical length equals to integer number of wavelengths. In the empty cylinder resonator due to symmetry the length of such a tra-

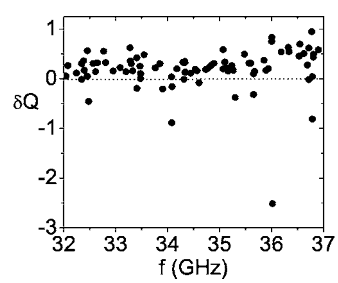

FIG. 9. The relative Q-factor deviation $\delta Q=(Q_{\text{empty}}-Q_{\text{inhom}})/Q_{\text{empty}}$ depends upon frequency f for the resonator filled with inhomogeneities. For the major part of resonances the relative Q-factor deviation is more than zero, i.e., their Q is less than for the empty resonator. The relative Q-factor deviation is less than zero for several nearest-neighbor resonances that have overlapped spectral lines (for the empty resonator) and split into separate resonances with inhomogeneities, i.e., the spectral line repulsion takes place here (see Fig. 8).

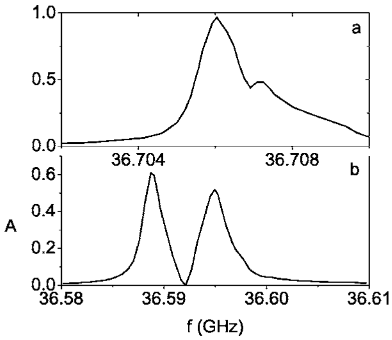

FIG. 10. The adjacent resonances with azimuthal and radial indexes $n=34$, $l=3$; $n=4$, $l=15$; in the empty resonator (a) and in the resonator filled by inhomogeneities (b). The normalization of the spectral lines amplitude was made on the maximal spectral line amplitude for the empty resonator.

jectory does not depend on its initial (final) point on the resonator side face. The given resonant frequency corresponds to a set of trajectories with the same optical length. The widths of spectral lines caused by ohmic loss in the resonator is the same for different trajectories for the given frequency. If the resonator is filled with small inhomogeneities optical length for various trajectories becomes slightly differ. Correspondingly, the resonant frequencies become also slightly different. Owing to small inhomogeneities the presence of a set of trajectories with close optical length leads to merging of spectral lines into one nonuniformly broadened spectral line. Broadening of this line consists of two components: one is the dissipative component due to ohmic loss in the resonator and the other one is nondissipative component due to merging of close spectral lines caused by random inhomogeneities.

Such physical understanding corresponds to experimental situation. If the resonator is excited by means of a point dipole with a wide directional diagram a set of different resonant trajectories are appeared simultaneously according to the ray treatment. These closed-loop trajectories constitute a statistical assemble by which the self-averaging is realized. Thus, the spectral line observed in the experiment is the result of the self-averaging process.

V. ACTIVE RESONATOR WITH RANDOM INHOMOGENEITIES

We also studied an active quasioptical resonator with random inhomogeneities. In comparison with the considered above passive resonator the active one is a self-excited oscillation system where the presence of random inhomogeneities affects on the excitation of resonator oscillations. For that we used the same resonator as mentioned above (Fig. 2) with a point Gunn diode inside. The microwave electrical field for TE mode was directed along a diode's axis. The diode dc power supply was implemented through a filter as a quarter-wave microwave isolator. Owing to that the spurious microwave radiation was prevented.

The quasioptical resonator with the Gunn diode is an active oscillator with distributed parameters. Near the threshold

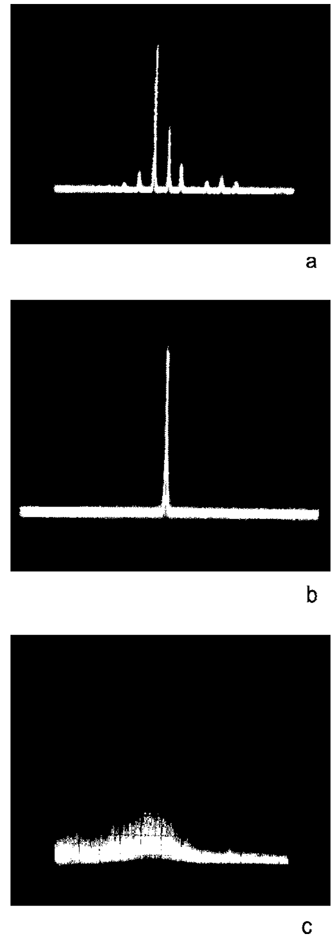

FIG. 11. Oscillogram of the 36 GHz generation obtained by a millimeter wave spectrum analyzer; (a) and (b) are for the empty resonator. Multifrequency generation near the generation threshold (a) and monofrequency generation much far from the generation threshold (b); (c) is chaotic generation near the generation threshold for the resonator filled with inhomogeneities. Monofrequency stable generation in the resonator with inhomogeneities has the similar view as in (b) much far from the generation threshold.

of excitation in such an oscillator with the empty resonator unstable multifrequency generation was detected [Fig. 11(a)]. We can explain such generation by frequency jumps between adjacent spectral lines with high quality factor. If the excitation threshold was highly exceeded monofrequency generation occurs [Fig. 11(b)], as a result of frequency competition. The active oscillator selects "itself" the only frequency to provide maximal regeneration factor [25].

The random inhomogeneities lead to effective rarefaction of the resonator spectrum. The number of adjacent spectral lines with high quality factor is reduced, and, as a result, multifrequency generation disappears. At small exceeding of the generation threshold the noise generation appears [Fig. 11(c)] in the resonator with inhomogeneities. At great exceeding of the generation threshold the stable monofrequency is observed likewise in [Fig. 11(b)]. Owing to inhomogeneities monofrequency generation possesses greater frequency stability, and a number of generating frequencies is reduced in the range of diode negative resistance. Figure

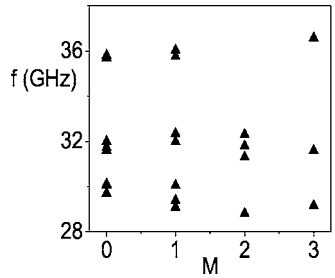

FIG. 12. The generating frequencies f depending on the number of inhomogeneities M. The empty resonator ($M=0$), the resonator quarter filled with inhomogeneities ($M=1$), the resonator half filled with inhomogeneities ($M=2$), the resonator entirely filled with inhomogeneities ($M=3$).

12 shows generating frequencies for the empty resonator and for the resonator with random bulk inhomogeneities.

VI. POSSIBLE APPLICATION TO NANOELECTRON SYSTEMS

Recently the great emphasis is the study of a new type of nanoelectron systems so-called zero-dimensional ones. The charge carrier motion in these systems has space restriction in all three dimensions. The peculiar example of zero-dimensional system is a quantum dot (QD). QD is a semiconductor area by the size of order of 10 nm with electron (hole) conductivity and restricted by potential barrier of outer area. Owing to finite charge carrier motion the energy spectrum in QD is discrete and the number of spectral levels due to small QD size is relatively few.

For the QD design an elegant method of QD array implementation has been designed. This method is based on the self-organization effect in strained double GaAs heterostructures [26–28]. The usage of QD array as an active medium permits to design lasers with high performance [20,29,30]. However, the self-organization process of QD array forming is difficult to manage. The random inhomogeneities that usually exist in heterostructure essentially affect this process. They lead to inhomogeneous broadening of spectral lines [31] and, correspondingly, to degradation of laser radiation quality.

The QD design method based on the self-organization effect is, in fact, an alternative one to the electron lithography. The level of electron lithography development does not permit to implement the ordered array of approximately the same QDs with enough small dispersion of their sizes. We propose another way to design a semiconductor laser system. This way presupposes the use of the same regular microscopic ordered array areas that can be implemented by lithography as an active laser medium. This proposal is based on the following. At present the technology of GaAs monocrystal with super high mobility and big length of phase coherence of charge carriers (that achieves 10 μm and moreover) is well developed. That is way, the system of microscopic regular areas with potential barrier on the boundary of each of them made from such materials can be implemented. Because of big length of phase coherence, carriers will take part in ballistic motion and reflect back from boundaries. The motion of charge carriers is similar to dynamics of billiard systems. Since this motion is described by a Schrödinger equation such a billiard system can be characterized as a quantum billiard (QB).

Due to finite motion of charge carrier, the electron spectrum of QB is discrete. At the same time it is quite dense, because of the QB size is much bigger than the wavelength of quasiparticles. This fact makes complicate the QB usage as an active system for the semiconductor laser, because the dense spectrum decreases the frequency stability of laser radiation. The frequency jumps appear easily at a small deviation of control parameters in the laser resonator with dense frequency spectrum. As a result, there is a problem of "rarefaction" of the spectrum: Can the dense QB spectrum be done much sparser without changing QB geometrical parameters?

Thus, based on our results we state that the use of the quasioptical cavity resonator filled with inhomogeneities gives the possibility of essential spectrum rarefaction and we can give the positive answer the question above. The quasioptical cavity resonator has dense and discrete frequency spectrum. The Maxwell equation describing electromagnetic oscillations in such a resonator coincides with corresponding scalar Schrödinger equation at definite conditions. All that gives an opportunity to use the quasioptical resonators as model objects to study spectral properties of QB. Similar modeling was done earlier for the study of the phenomena that is relevant to quantum chaos [23,24,32,33].

The inhomogeneous quasioptical cavity millimeter wave resonator (passive and active) can serve as a model of semiconductor quantum billiard. Based on our results we suggest to use QB with spectrum rarefied by random inhomogeneities as an active system of semiconductor laser.

VII. CONCLUSION

The statistical spectral theory of the quasioptical cavity resonator filled with random dielectric inhomogeneities was developed. We showed that the presence of inhomogeneities leads to the broadening and shift of spectral lines. It is found that the physical nature of broadening and shift of spectral lines is relevant to intermode scattering. The scattering effect for the given spectral line essentially depends on a frequency distance between of it and adjacent ones and is sharply decreased for bigger distances. Under the influence of random inhomogeneities the original spectrum modification occurs that can be interpreted as spectrum rarefaction. The spectrum is rarefied because of solitary spectral lines are not practically subjected to the influence of inhomogeneities. The quality factor of such lines and, correspondingly, their intensities stay high at the resonator excitation. The intensity of adjacent lines broadened under the influence of inhomogeneities is essentially reduced. Owing to that, at the great number of inhomogeneities the resonator spectrum is rarefied, i.e., few solitary high quality factor spectral lines prevail in the spectrum.

Theoretical prediction of the broadening and shift of spectral lines and spectrum "rarefaction" are subject to experimental check. For that purpose we experimentally studied at 8-millimeter wave range the spectrum of the quasioptical cavity resonator filled with random small-scattered bulk inhomogeneities. These inhomogeneities were styrofoam particles with smaller size than the operating wavelength. We have found out that such inhomogeneities lead to broadening and shift of spectral lines. As experiment showed that the maximum of their influence was only on nearest-neighbor spectral lines. The solitary lines, according to our theory, were subjected to this influence in much smaller degree. We detected the effect of stochastic spectrum "rarefaction." It was proved that main mechanism of broadening and shift of spectral lines is relevant to inhomogeneities intermode scattering.

In addition we studied the chaotic properties of oscillations in our resonator. It is found out that the empty resonator has IF intervals distribution similar to the Poisson distribution that is typical to the spectrum with noncorrelated IF intervals. Even at small number of inhomogeneities the resonator spectrum has random part that increases in proportion to the number of inhomogeneities in the resonator. At that IF intervals distribution in random inhomogeneous resonator is described by the Brody and Berry-Robnik distributions of IF intervals.

We obtained the results concerning the influence of bulk inhomogeneities on the process of generation in a self-excitation system. The self-excitation system was a quasi-optical millimeter wave cavity resonator containing inhomogeneities with an active element as a Gunn diode. We detected that inhomogeneities lead to the essential "rarefaction" of the spectrum and create conditions for monochromatic stable generation in self-excitation system.

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